Boulware state in exactly solvable models of 2D dilaton gravity

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We discuss self-consistent geometries and behavior of dilaton in exactly solvable models of 2D dilaton gravity, with quantum fields in the Boulware state. If the coupling $H(\phi)$ between curvature and dilaton ϕ is non-monotonic, backreaction can remove the classical singularity. As a result, an everywhere regular star-like configuration may appear, in which case the Boulware state, contrary to expectations, smooths out the system. For monotonic $H(\phi)$ exact solutions confirm the features found before with the help of numerical methods: the appearance of the bouncing point and the presence of isotropic singularity at the classically forbidden branch of the dilaton.

Key words: dilaton, Boulware state, two-dimensional gravity

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I. INTRODUCTION

The backreaction of quantum field on a black hole is significant at late stages of evaporation due to the Hawking effect and, thus, affects a final fate of a black hole. As far as a static black hole is concerned, the role of backreaction on the background of a non-extremal black hole depends crucially on the state of quantum fields. In the Hartle-Hawking state the quantum stresses are bounded everywhere, so that small backreaction shifts parameters of a black hole but does not change its properties qualitatively. The situation changes drastically for the Boulware state that corresponds to the Minkowski vacuum at infinity. As is well known, near the horizon such quantum stresses diverge, so in general relativity a

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classical regular horizon is destroyed in this case. The question arises, what happens to the quantum-corrected geometry whose classical limit corresponds to a black hole. In doing so, one cannot consider the behavior of quantum stresses in a fixed background but, instead, should solve field equations in a self-consistent way.

Motivation to study such geometries comes from different directions. As is pointed out in [1], [2] this concerns analogy with brane-world physics in the 5D world, where the problem of finding static black hole solutions is mapped to the problem of finding self-consistent solutions in the 4D world with quantum fields in the Boulware state [3], [4]. Another line of motivation is, in our view, connected with the physically important problem of the fate of Cauchy horizons which are present, say, in classical charged black holes. It is believed that it is classically unstable (see, e.g. Sec. 14.3.2 of the monograph [5] and references therein). Meanwhile, one can expect also another kind of instability here due to infinite quantum backreaction of massless fields near such horizons. This resembles the situation with the Boulware state but the latter is certainly more simple, so its study could help us to understand better what happens in the vicinity of Cauchy horizons. Apart form this, the Boulware state as such arises in rather simple and natural way, being singled out by the condition that the quantum stresses vanish at infinity, so one is led to elucidate what happens to self-consistent solutions whose classical analogues were black holes. In particular, it is not clear in advance whether we will obtain a singular horizon or regular configuration without a horizon. This probelms

Recently, the problem under discussion for the Schwarzschild black hole was tackled in [1], [2] where it was found that the metric component g_{00} and the areal radius cease to be monotonic functions of the coordinate and acquire bouncing points. Similar features seem to occur also in higher-dimensional theories. These conclusions alter the general picture qualitatively in that quantum-corrected geometries whose classical counterparts were singular near a classical would-be horizon, become regular there but generate a branching point beyond it and a singularity having no classical analogue.

Meanwhile, the validity of the results obtained in [1], [2] is obscured by the assumptions that cannot be controlled easily. The corresponding approach exploits spherically-symmetrical reduction from the four-dimensional (4D) theory to the effective two-dimensional (2D) one. In doing so, all contributions to the stress-energy tensor except the s-wave one are ignored that, in general, is not quite good approximation [6]. Further, simplification was used in [1], [2] based on the replacement of the action of scalar field by its Polyakov-Liouville form. It was motivated by the properties of the wave equation near the usual non-extremal regular horizon. However, it is not quite clear how this approximation and the distinction between 2D and 4D theories as a whole affect the results in the entire region and, in addition, the horizon itself can become singular. Even after all these approximations the field equations remain quite complicated and one is led to the implementation of numerical methods.

Fortunately, there is the situation when field equations admit self-consistent exact solutions with backreaction taken into account. This is 2D dilaton gravity considered as a true 2D theory from the very beginning, with minimal quantum fields described by the Polyakov-Liouville action without any additional assumptions. Remarkably, such theories admit exactly solvable models provided the action coefficients obey some relation. In the present paper we show some qualitative features of the Boulware state using simple and known exactly solvable models and their generalizations. In particular, we trace how backreaction can destroy classical horizons and/or singularities and create new ones of pure quantum nature. Some results turn out to be similar to those found in [1], [2] while some seem not to have counterparts in general relativity (but may have them in 4D dilaton gravity). In a more formal way, one can pose the problem of finding self-consistent solutions of field equations under two conditions: (i) quantum stresses vanish at infinity, (ii) in the classical limit there exists a classical horizon. Then we will see that there are two main cases: either the singular horizon exists (in which case we consider the type of singularity in more detail) or the horizon disappears at all giving way to a star-like configuration. (We must make reservation that we do not consider black hole solutions in higher curvatures theories - see, e.g., [7].)

II. BASIC EQUATIONS

Consider the action

$$I = I_0 + I_{PL}, \tag{1}$$

where

$$I_0 = \frac{1}{2\pi}\int_M d^2x\sqrt{-g}[F(\phi)R + V(\phi)(\nabla\phi)^2 + U(\phi)] \tag{2}$$

and I_{PL} is the Polyakov-Liouville action incorporating effects of quantum fields minimally coupled to the background. We assume that backreaction is due to a multiplet of N fields and neglect quantum backreaction of dilaton itself in the large N approximation, as usual (see. e.g. Sec. 3.7 of [8]). In doing so, $\hbar \to 0$ $N \to \infty$ in such a way that the quantum coupling parameter $\kappa = \frac{\hbar N}{24}$ remains finite. Let us consider the potential in the form $U = 4\lambda^2 \exp(-2\phi)$ that arises naturally in string-inspired models. If the action coefficients obey the condition

$$V = -2(\frac{dF}{d\phi} + \kappa), \tag{3}$$

the field equation have exact static solutions which admit complete classification [9] (see also references therein). Then we may borrow directly explicit expressions from eqs. (52), (53) of [9] where the case of Boulware state for the asymptotically flat metric is contained implicitly among a variety of exact solutions. Hereafter, we use the quantum-corrected coefficient $H(\phi) = F - \kappa \ln U$ instead of the original one $F(\phi)$. In conformal coordinates the metric takes the form

$$ds^2 = g(-dt^2 + d\sigma^2), \tag{4}$$

$$R = -\frac{2\lambda^2}{g}\frac{d^2\phi}{dy^2}, \; y \equiv \lambda\sigma \tag{5}$$

and we have

$$H(\phi) = f(y), \; f(y) \equiv e^{2y} - \kappa y + H_0, \; g = e^{2y+2\phi}. \tag{6}$$

$$T_1^{1(PL)} = -\frac{1}{4\pi}e^{-2y}\{2\alpha + \kappa(\frac{\alpha}{2\lambda} - 2\lambda e^{2y})2H^{'-1}[-4\lambda + (\frac{\alpha}{2\lambda} - 2\lambda e^{2y})2H^{'-1}]\}, \tag{7}$$

Here R is the Riemann curvature, $T_\nu^{\mu(PL)}$ is the stress-energy of quantum fields, $\kappa = \frac{N}{24}$ is the quantum coupling parameter, N is a number of field in a multiplet. As is well known, the trace $T_\mu^{\mu(PL)} = \kappa\frac{R}{\pi}$. We also assume that at the right infinity $y \to \infty$ there is a linear dilaton vacuum, $\phi = -y$, $g \to 1$. Correspondingly, we assume that, asymptotically at $\phi \to -\infty$, $H \simeq \exp(-2\phi)$. The Boulware state implies that $T_1^{1(PL)} \to 0$ at $y \to \infty$. It is easy to check by substitution into (7) that this is achieved by the choice $\alpha = 2\lambda^2\kappa$.

In the classical limit $\kappa = 0$. Then, at $y \to -\infty$ and finite $\phi = \phi_h$ we have $g \to 0$, so this is a black hole horizon, $H_0 = H(\phi_h)$. If $H = e^{-2\phi}$, we obtain an usual CGHS black hole [10] with $g = \frac{e^{2y}}{e^{2y}+H_0} = 1 - e^{2(x_h - x)}$, hereafter $x = \int dy g$ is the Schwarzschild-like coordinate, x_h is its horizon value.

It is essential that, whatever the function $H(\phi)$ be, the function $f(y)$ has an universal form, independent of the particular dilaton gravity model. At $y\to\infty$ and $y\to-\infty$ the function $f\to\infty$. It has one local minimum at $y=y_0=\frac{1}{2}\ln\frac{\kappa}{2}$, $f(y_0)=H_0+\frac{\kappa}{2}(1-\ln\frac{\kappa}{2})$.

III. BEHAVIOR OF METRIC AND DILATON

In what follows we will be interested only in such configurations when the classical limit ($\kappa=0$) corresponds to a black hole. This excludes the situation when $H(\phi_0)>f(y_0)$ where ϕ_0 is the point of minimum of H (if it exists). Indeed, in this case, when y attains the point y_1 where $f(y_1)=H(\phi_0)$, eq. (6) gives us that $H''(\phi_0)\frac{(\phi_0-\phi)^2}{2}=f'(y_1)(y-y_1)$, the metric function g is finite, the curvature $R\sim H'^{-3}\sim(y-y_1)^{-\frac{3}{2}}$ diverges, so we have a naked time-like singularity which persists also in the limit $\kappa\to0$. The horizon does not exist at all, so this situation has nothing to do with the Boulware state. In a similar way, if H approaches its limiting value at $\phi\to\infty$ asymptotically and $H(\infty)=f(y_0)$, a semi-infinite throat arises instead of a black hole [11] that is beyond the scope of our paper. Therefore, we restrict ourselves by the following cases.

1) $H(\phi)$ is monotonic and unbounded. The typical example is

$$H=e^{-2\phi}-a\kappa\phi,\ U=4\lambda^2e^{-2\phi},\ a>0. \tag{8}$$

The dilaton field $\phi=-\infty$ at $y=\infty$. If, further, y decreases, classically the dilaton field grows monotonically up to the horizon where $y=-\infty$ and $\phi=\phi_h$ is finite, $H(\phi_h)=H_0$. However, if $\kappa\neq0$, the bouncing point appears in the solution at $y=y_0$. At this point the derivative $\frac{d\phi}{dy}$ changes its sign, so ϕ again decreases to $-\infty$ as $y\to\infty$. At the point y_0 the metric function $g(y_0)=\frac{\kappa}{U(y_0)}$. The fact that $\left(\frac{d\phi}{dy}\right)_{y=y_0}=0$ while the metric function $g\neq0$ entails that $(\nabla\phi)^2=0$. This is nothing other than the 1+1 analogue of the apparent horizon $(\nabla r)^2=0$ which does not coincide now with the event horizon. In the limit $\kappa\to0$ we have $y_0\to-\infty$, $g(y_0)\to0$ and a classical event horizon restores. For a given small but non-zero value of κ the whole region $y<y_0$ has no classical counterpart.

The metric function in (6) can be rewritten also in the form $g=e^{2\rho}$, $\rho=y+\phi$. At $y=y_0$ $\frac{d\rho}{dy}=1$, and we have $\left(\frac{dr}{d\rho}\right)_{y=y_0}=0$, where $r=\exp(-\phi)$ also in qualitative agreement with [1], [2].

At $y\to-\infty$ the dilaton field for the model (8) $\phi=-\frac{1}{2}\ln|y|$, the metric function

$g \sim \frac{\exp(2y)}{|y|} \to 0$, the curvature

$$R \sim -\frac{\lambda^2\kappa^2}{r^2}\exp(2\frac{r^2}{\kappa}) = -\frac{\lambda^2\kappa}{|y|}\exp(2\,|y|), \tag{9}$$

where $r \to \infty$, so we have a singular horizon. Thus, the non-zero quantum coupling parameter κ is essential in that $\lim_{y\to-\infty} R$ diverges. The features of the solution discussed above in case 1) are qualitatively similar to those found in [1], [2].

2) $H(\phi) \to \infty$ at $\phi \to \pm\infty$ and has one minimum at $\phi = \phi_0$. The typical example is

$$H = e^{-2\phi} + c\phi,\, U = 4\lambda^2 e^{-2\phi},\, c = b\kappa > 0. \tag{10}$$

The value $b = 1$ corresponds to the so-called RST model [12], [13]. For a given y, there exist two branches of the solution $\phi(y)$. For definiteness, we choose the branch that corresponds to the linear dilaton vacuum $\phi = -y$ at the right infinity. Then, depending on the value of H_0, we have the following subcases.

a) $H(\phi_0) < f(y_0)$. Then, as y changes from plus to minus infinity, the point representing our system on the curve $H(\phi)$ moves from $\phi = -\infty$ at $y = \infty$ until the point y_0 and back to $\phi = -\infty$ for $y < y_0$ along the same branch of the function $H(\phi)$, so the situation is similar to case 1, the presence of the second branch of $H(\phi)$ does not manifest itself.

b) The most interesting case arises when $\phi_0 = \phi(y_0)$, so that $H(\phi_0) = f(y_0)$. Then near y_0

$$H''(\phi_0)\frac{(\phi-\phi_0)^2}{2} = f''(y_0)\frac{(y-y_0)^2}{2}, \tag{11}$$

the derivative $\frac{d\phi}{dy} = \frac{f'(y)}{H'} = \frac{f''(y_0)(y-y_0)}{H''(\phi-\phi_0)} \to -\sqrt{\frac{f''(y_0)}{H''(y_0)}} < 0$.Thus, the turning point for the dilaton disappears, near y_0 the dilaton can be expanded into series with respect to $y - y_0$, the metric function g and curvature being finite.

It is worth reminding that, for a given H_0, the singularity of the classical CGHS black hole was situated at $\phi \to \infty$, $x \to -\infty$ (the proper distance being finite), $R \sim \exp(-x)$. Inclusion of backreaction shifts the singularity to the finite value ϕ_0. It was shown in [14] that in the Hartle-Hawking state of the RST model the curvature $R \sim (x_h - x)^{-3/2}$, if (in our notations) $H(\phi_0) < f(y_0)$. The singularity becomes milder, $R \sim (x_h - x)^{-1/2}$, if $H(\phi_0) = f(y_0)$, but still persists. Meanwhile, as we see, in the Boulware state the singularity at $\phi = \phi_0$ disappears at all. In this sense, contrary to expectations, the Boulware turns out to be "more regular" than the Hartle-Hawking one!

Thus, $\phi(y)$ continues to diminish when we pass the point ϕ_0 in the direction of increasing the dilaton field and move to the branch $\phi > \phi_0$.Then, we have from (6) and (10) that, asymptotically at $y \to -\infty$ the dilaton $\phi \sim -\frac{y}{b}$ and

$$g \sim \exp[2\,|y|\,(\frac{1}{b}-1)],\; R \sim \exp[-2\,|y|\,q], \tag{12}$$

where $q = \frac{1}{b}$ for $b \leq 1$ and $q = \frac{2}{b} - 1$ for $b \geq 1$. The properties of the spacetime depend crucially on the value of b.

b1) $b < 1$. Then, $g \to \infty$, the curvature $R \to 0$. In terms of the Schwarzschild-like coordinate the metric function $g \sim |x|$. Thus, the metric has the asymptotically Rindler form and, in this sense, corresponds to an accelerated observer. However, there is no sense to speak about the acceleration horizon at $x = 0$ since this form of the metric is valid for $|x| \to \infty$ only. One can check that in (7) $T_1^{1(PL)} \to 0$ because of the factor $g^{-1} \to 0$.

b2) $b = 1$. This is the RST model [12], [13]. Then $g \to const$, $R \to 0$, so we have the Minkowski spacetime at both infinities. Now, $T_1^{1(PL)} \to 0$ because of vanishing braces in (7).

b3) $1 < b \leq 2$. Then $g \to 0$ (horizon) and $R \to 0$ or $R \to const$. The remarkable feature of these solutions consists in that the geometry near the horizon is regular but quantum stresses $T_1^{1(PL)}$ diverge. There exists the whole class of such models described in more detail in [16].

b4) $b > 2$. Then $g \to 0$ but $R \to \infty$, so we have a singular horizon.

Up to now, the parameter κ played the double role in that it changed the behavior of both functions $H(\phi)$ and $f(y)$. For the string inspired models like (10) in the classical limit $\kappa = 0$ the function $H = e^{-2\phi}$ is monotonic but H has a minimum for $\kappa \neq 0$. In a similar way, the function $f(y) = e^{2y} + H_0$ is monotonic classically but acquires the minimum at y_0 when $\kappa \neq 0$. Meanwhile, it makes sense to consider a somewhat different situation in which the parameter $c > 0$ in (10) has a non-vanishing classical limit (for example, we may take $c = d + \kappa b$ where d does not contain κ). Then, account for backreaction changes the behavior of $f(y)$ qualitatively but only slightly changes the value of the coefficient c in $H(\phi)$. This allows us to compare the properties of the Boulware state to those of the classical configuration in a more direct way. As case 2a) is similar to case 1) with the monotonic $H(\phi)$, we restrict ourselves to case 2b).

If the condition $H(\phi_0) = f(y_0)$ is satisfied, in the classical limit $y_0 \to -\infty$ and eq. (6) gives rise to $H''(\phi_0)\frac{(\phi-\phi_0)^2}{2} = e^{2y} \sim x - x_h$, whence $\phi = \phi_0 - \sqrt{\frac{2}{H''(\phi_0)}}e^y[1+O(e^y)]$. As a result, the curvature $R \sim e^{-y} \sim (x_h - x)^{-1/2}$ like in the Hartle-Hawking state of the RST model [14]. In the quantum case the behavior of the dilaton is determined near y_0, in the main approximation, by a quite different equation (11). Keeping also the third terms of the Taylor expansion near ϕ_0 one finds that $3\frac{d^2H}{d\phi^2}(\phi_0)\left(\frac{d^2\phi}{dy^2}\right)_{y=y_0} = \left(\frac{d\phi}{dy}\right)^{-1}_{y=y_0}\frac{d^3f(y_0)}{dy^3} - \frac{d^3H}{d\phi^3}(\phi_0)\left(\frac{d\phi}{dy}\right)^2_{y=y_0}$. Taking into account that $\frac{d^2f(y_0)}{dy^2} \sim \frac{d^3f(y_0)}{dy^3} \sim \kappa$ while $\frac{d^2H}{d\phi^2}(\phi_0)$ and $\frac{d^3H}{d\phi^3}(\phi_0)$ do not contain κ, $g \sim e^{2y_0} \sim \kappa$, we obtain from (5) that the curvature is finite due to backreaction and in the limit $\kappa \to 0$ $R(y_0) \sim \frac{1}{\sqrt{\kappa}}$.

Near $y \to \infty$ eq. (12) is valid in which we must replace $b \to \frac{c}{\kappa}$. If c is finite but $\kappa \to 0$, $b \gg 1$, so that case b4) is realized and a singular horizon appears.

To conclude this section, we would like to point out that the coupling of dilaton to curvature is described in the quantum-corrected action by the coefficient H instead of F in the pure classical case (2). Therefore, the effective gravitation-dilaton constant $\gamma \sim H^{-1}$. As a result, it follows from (8), (10) that even in the region where the dilaton ϕ itself infinitely grows, this constant remains finite and, moreover, tends to zero like ϕ^{-1}. By contrast, in the pure classical action the corresponding constant $\gamma \sim F^{-1}$ infinitely grows. In this sense, account for backreaction automatically expands the region of validity of semiclassical approximation.

IV. SUMMARY

The advantage of the approach under discussion is that we solve the problem analytically and, moreover, have at our disposal exact solutions which are quite simple and convenient for analysis. We considered two main situations. 1) The curvature coupling $H(\phi)$ is monotonic. Then we demonstrated such features as the appearance of the bouncing point in the solution, transformation of the classical regular event horizon into the apparent one and the appearance of the singular horizon having no classical analogue. All these features are qualitatively similar to those found in [1], [2], although the concrete gravitational-dilaton actions do not coincide.

However, there is also a quite different case 2b) which seems to have no analogue in general relativity and in the effective 2D theory considered in [1], [2]. If $H(\phi)$ has a minimum ϕ_0 where $H(\phi_0) = f(y_0)$, the singular horizon typical of the Hartle-Hawking state or the classical black hole disappears in the Boulware state, the point ϕ_0 being regular due to backreaction.

The new part of the manifold opens behind it which, classically, was unreachable from infinity. In doing so, there is neither an apparent nor an event horizon in the vicinity of the point y_0. A singular horizon may appear to the left from this point but it is situated at a finite distance from it and has pure quantum origin. In doing so, the dilaton value $\phi \to \infty$ while the typical classical horizon has a finite value ϕ_h on the horizon. Apart from this, the region beyond the point y_0 may represent a star-like configuration with the asymptotic left infinity of two kinds: either the Minkowski metric or the Rindler one. In addition, we would like to pay attention to the previous observation [15], [16] that quantum stresses in the Boulware state that destroy the classical would-be horizon completely may also lead to the appearance of a new regular (!) horizon of pure quantum origination. We see that the Boulware state may not only bring about singularities into a classical black hole background but, vice versa, smooth out the singularities of the classical counterpart and create everywhere regular configurations.

Thus, quantum terms not simply dominate near the classical would-be horizon but may change the structure of the spacetime qualitatively. It is of interest to trace whether similar effects can occur in 4D dilaton theory.

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